Effect of shear on persistence in coarsening systems

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We analytically study the effect of a uniform shear flow on the persistence properties of coarsening systems. The study is carried out within the anisotropic Ohta-Jasnow-Kawasaki (OJK) approximation for a system with nonconserved scalar order parameter. We find that the persistence exponent θ has a nontrivial value: $\theta=0.5034\ldots$ in space dimension $d=3$, and $\theta=0.2406\ldots$ for $d=2$, the latter being exactly twice the value found for the unsheared system in $d=1$. We also find that the autocorrelation exponent λ is affected by shear in $d=3$ but not in $d=2$.

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I. INTRODUCTION

The phenomenon of persistence in nonequilibrium systems has attracted considerable interest in recent years [1], both theoretically [2–4] and experimentally [5–9]. The persistence probability $P_0(t)$ of a fluctuating, spatially homogeneous nonequilibrium field is the probability that the field $X(t)$ at a given space point has not changed sign up to time t. This probability typically decays as a power law $P_0(t)\sim t^{-\theta}$ at late times, where the persistence exponent θ has in general a nontrivial value. Persistence has been studied in a considerable number of systems such as simple diffusion from random initial conditions, phase-ordering kinetics, fluctuating interfaces, and reaction-diffusion processes [1].

Experiments to determine persistence exponents have been carried out in the context of breath figures [5], liquid crystals [6], soap froths [7], diffusion of Xe gas in one dimension [8], and fluctuating monatomic steps on a metal/semiconductor adsorption system Si-Al surface [9]. Many of these cases are examples of coarsening phenomena, where a characteristic length scale increases with time as the system relaxes toward an equililibrium that it attains only after infinite time in the thermodynamic limit. The experimental results are generally in good quantitative agreement with (exact or approximate) theoretical predictions.

A classic example of a coarsening phenomenon is the dynamics of phase ordering, where a system is quenched from a disordered high-temperature phase into an ordered low-temperature phase. In the simplest case of a two-phase system, domains of the two equilibrium phases form and grow with time. The characteristic length scale at a given time is the typical scale of the domain structure that has formed at that time. The coarsening dynamics is usually characterized by a form of dynamical scaling, in which the system looks statistically similar at different times apart from an overall change of scale [10].

Recently there has been interest in the effect of shear in a variety of systems such as macromolecules, binary fluids, and self-assembled fluids [11]. Shear introduces anisotropy into the spatial structure. For systems undergoing phase ordering in the presence of shear, the domain growth becomes anisotropic and this results in different growth exponents for the structure along and perpendicular to the flow. At present it is not clear whether shear leads to a stationary steady state, or whether domain growth proceeds indefinitely at asymptotically large times [12]. Shear may also induce phase transitions: For example, shear-induced shift of the phase transition temperature in the microphase separation of diblock copolymers has been observed [13].

In this paper we analytically study the effect of an imposed uniform shear flow on persistence for the simplest case of a nonconserved scalar order parameter. We exploit a version of the Ohta-Jasnow-Kawasaki (OJK) approximation in phase-ordering kinetics [10], modified to account for the anisotropy induced by the shear [14]. Persistence is defined here as the probability that a point *comoving with the flow* has remained in the same phase up to time t. We employ an approach called the independent interval approximation (IIA) which has been successfully used to obtain rather accurate values for persistence exponents in unsheared systems [1]. This procedure assumes that the intervals between zeros of the process $X(t)$ are statistically independent when measured in the mapped time variable $T=\ln t$. We find that the persistence exponent θ is nontrivial and dimensionality dependent. For $d=3$ we find $\theta\simeq 0.5034$, compared to $\simeq 0.2358$ in the unsheared case [3], while $\theta\simeq 0.2406$ for $d=2$ compared to $\theta\simeq 0.1862$ without shear [3]. Remarkably, the value of θ in $d=2$ is exactly twice the value obtained for the unsheared system in $d=1$ [3] using similar methods. There is a technical subtlety in $d=2$ which requires a careful definition of the persistence probability. In both $d=2$ and $d=3$ the shear increases the persistence exponent.

The paper is organized as follows. In the next section the OJK theory is introduced and the autocorrelation function, which is a necessary input to the IIA calculation, is obtained for $d=3$ and $d=2$. Section III contains a brief outline of the IIA, the results of which for the sheared problem are presented in Sec. IV. Concluding remarks are given in Sec. IV.

II. OJK THEORY

We consider a nonconserved scalar order parameter $\phi(\vec{x},t)$ evolving via the time-dependent Ginzburg-Landau equation [10]

$$\frac{\partial \phi(\vec{x},t)}{\partial t} = \nabla^2 \phi(\vec{x},t) - V'(\phi), \quad (1)$$

where $V(\phi)$ is a symmetric double-well potential. The assumption that the thickness ξ of the interface separating the domains is much smaller than the size of the domains allows one to write an equation of motion for the interface, called the Allen-Cahn equation [15]. The velocity v of the interface is proportional to the local curvature and given by

$$v(\vec{x},t) = -\vec{\nabla} \cdot \vec{n}(\vec{x},t), \quad (2)$$

where $\vec{n}(\vec{x},t)$ is the unit vector normal to the interface, defined in the direction of increasing order parameter. The normal vector can be written in general as

$$\vec{n}(\vec{x},t) = \frac{\vec{\nabla} m(\vec{x},t)}{|\vec{\nabla} m(\vec{x},t)|}, \quad (3)$$

where $m(\vec{x},t)$ is the smooth field that has the same sign as the order parameter ϕ and vanishes at the interfaces (where the order parameter vanishes). It is easier to work with an equation of motion for $m(\vec{x},t)$ than for $\phi(\vec{x},t)$, an idea that is exploited in the OJK theory [16].

By considering a frame locally comoving with the interface, with a space-uniform shear in the y direction and flow in the x direction (i.e., the fluid velocity profile is given by $\vec{u}=\gamma y \vec{e}_x$), where γ is the constant shear rate and $\vec{e}_x$ is the unit vector in the flow direction, the OJK equation for the field $m(\vec{x},t)$ becomes [14]

$$\frac{\partial m(\vec{x},t)}{\partial t} + \gamma y \frac{\partial m(\vec{x},t)}{\partial x} = \nabla^2 m(\vec{x},t) - \sum_{a,b=1}^{d} D_{ab}(t) \frac{\partial^2 m(\vec{x},t)}{\partial x_a \, \partial x_b}, \quad (4)$$

where

$$D_{ab}(t) = \langle n_a n_b \rangle, \quad (5)$$

and $\langle \cdots \rangle$ denotes average over initial conditions (or, equivalently, over space). The correct equation for m involves an unaveraged D_{ab}, but the equation is then nonlinear and intractable. The essence of the OJK approximation is the replacement of the product $n_a n_b$ by its average. For an isotropic system this gives, by symmetry $D_{ab}=\delta_{ab}/d$, and the equation for m reduces to the diffusion equation. For the anisotropic sheared system, however, $D_{ab}(t)$ has to be determined self-consistently [14]. From Eq. (5), it follows that

$$\sum_{a=1}^{d} D_{aa}(t) = 1. \quad (6)$$

In k space, Eq. (4) can be written as

$$\frac{\partial m(\vec{k},t)}{\partial t} - \gamma k_x \frac{\partial m(\vec{k},t)}{\partial k_y} = \left(- \sum_{a=1}^{d} k_a^2 + \sum_{a,b=1}^{d} D_{ab}(t) k_a k_b \right) m(\vec{k},t). \quad (7)$$

A. Case $d=3$

We now consider the above equation in dimension $d=3$ and solve it via the following change of variables [14]:

$$(k_x, k_y, k_z, t) \rightarrow (q_x, q_y - \gamma k_x \tau, q_z, \tau), \quad (8)$$

with the introduction of an equivalent field

$$\mu(\vec{q},\tau) = m(\vec{k},t). \quad (9)$$

The left-hand side of Eq. (7) now becomes $\partial \mu / \partial \tau$ and as a result Eq. (7) can be integrated directly to give (after transforming back to original variables)

$$m(\vec{k},t) = m(k_x, k_y + \gamma k_x t, k_z, 0) \exp\left[-\frac{1}{4} \sum_{ab=1}^{3} k_a M_{ab}(t) k_b \right], \quad (10)$$

with nonvanishing matrix elements

$$M_{11}(t) = R_{11}(t) + 2\gamma t R_{12}(t) + \gamma^2 t^2 R_{22},$$
$$M_{12}(t) = R_{12}(t) + \gamma t R_{22}(t),$$
$$M_{22}(t) = R_{22}(t),$$
$$M_{33}(t) = R_{33}(t), \quad (11)$$

where

$$R_{11}(t) = 4\int_0^t dt' \{[1 - D_{11}(t')] + 2\gamma t' D_{12}(t') + \gamma^2 t'^2 [1 - D_{22}(t')]\},$$
$$R_{12}(t) = 4\int_0^t dt' \{- D_{12}(t') - \gamma t' [1 - D_{22}(t')]\},$$
$$R_{22}(t) = 4\int_0^t dt' \{1 - D_{22}(t')\},$$
$$R_{33}(t) = 4\int_0^t dt' \{1 - D_{33}(t')\}. \quad (12)$$

Due to the symmetry of the original OJK Eq. (4), the terms R_{13}, R_{23}, M_{13}, and M_{23} all vanish. The assumption that the initial condition $m(\vec{k},0)$ has a Gaussian distribution, appropriate to a quench from the high-temperature phase, is used throughout the paper.

In order to use the IIA to investigate the persistence properties of the coarsening system, it is first necessary [1,3] to compute the autocorrelation function of the rescaled field $X(t)=m(\vec{x},t)/\langle [m(\vec{x},t)]^2 \rangle^{1/2}$, which is constructed to have unit variance, using the initial correlator

$$\langle m(\vec{x},0) m(\vec{x}',0) \rangle = \Delta \delta^d(\vec{x} - \vec{x}'). \quad (13)$$

The quantity $\langle [m(\vec{x},t)]^2 \rangle^{1/2}$ can easily be evaluated to give

$$\langle [m(\vec{x},t)]^2 \rangle^{1/2} = \left[\frac{\Delta}{(2\pi)^{3/2}} \frac{1}{\sqrt{\text{Det}\, M(t)}} \right]^{1/2}. \tag{14}$$

Turning now to the two-time correlator of $X(t)$, we recall that we want to calculate this correlator not at a fixed point in space, but at a point that is advected with the shear flow. Due to the shear, the field at the space-time point $(x+\gamma y t_1, y, z, t_1)$ at time t_1 will be at the space-time point $(x+\gamma y t_2, y, z, t_2)$ at time t_2. The autocorrelation function $a(t_1,t_2)=\langle X(t_1)X(t_2)\rangle$ is therefore given by

$$a(t_1,t_2) = \left[\frac{(2\pi)^3}{\Delta^2} \sqrt{\text{Det}\, M(t_1) \text{Det}\, M(t_2)} \right]^{1/2} \times \langle m(x+\gamma y t_1, y, z, t_1) m(x+\gamma y t_2, y, z, t_2) \rangle. \tag{15}$$

The next step is to evaluate the term $\langle \cdots \rangle$ in the above equation. We note that average over initial conditions in k-space implies

$$\begin{aligned} &\langle m(k_x, k_y+\gamma k_x t_1, k_z, 0) m(k_x', k_y'+\gamma k_x' t_2, k_z', 0)\rangle \\ &= (2\pi)^d \Delta\, \delta(k_x+k_x')\, \delta(k_y+\gamma k_x t_1 + k_y' + \gamma k_x' t_2) \\ &\times \delta(k_z+k_z'). \end{aligned} \tag{16}$$

Using Eqs. (10) and (16) we can evaluate the term

$$\begin{aligned} \langle m(1) m(2) \rangle &= \Delta \sum_k \exp\left[-\frac{1}{2} \sum_{a,b=1}^{3} k_a B_{ab} k_b \right] \\ &= \frac{\Delta}{(2\pi)^{3/2}} \frac{1}{\sqrt{\text{Det}\, B(t_1,t_2)}}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} B_{11}(t) &= [M_{11}(t_1) + M_{11}(t_2) + \gamma^2 M_{22}(t_2) \times (t_2-t_1)^2 \\ &\quad - 2\gamma (t_2-t_1) M_{12}(t_2)]/2, \\ B_{12}(t) &= [M_{12}(t_1) + M_{12}(t_2) - \gamma (t_2-t_1) M_{22}(t_2)]/2, \\ B_{22}(t) &= [M_{22}(t_1) + M_{22}(t_2)]/2, \\ B_{33}(t) &= [M_{33}(t_1) + M_{33}(t_2)]/2, \\ B_{13}(t) &= B_{23}(t) = 0. \end{aligned} \tag{18}$$

The arguments 1 and 2 in Eq. (17) denote space-time points $(x+\gamma y t_1, y, z, t_1)$ and $(x+\gamma y t_2, y, z, t_2)$ respectively. The problem is now reduced to evaluating the determinants of the matrices $M(t)$ and $B(t_1,t_2)$, as the autocorrelation function $a(t_1,t_2)$ can now be written as

$$a(t_1,t_2) = \frac{[\text{Det}\, M(t_1) \text{Det}\, M(t_2)]^{1/4}}{\sqrt{\text{Det}\, B(t_1,t_2)}}. \tag{19}$$

The terms $M_{ab}(t)$ cannot be computed explicitly for general t; only in the scaling limit (i.e., $t\to\infty$) can one make progress. In this limit it can be shown that (to leading order for large t) [14]

$$\begin{aligned} M_{11}(t) &= \frac{4}{15} \gamma^2 t^3, \\ M_{12}(t) &= \frac{2}{5} \gamma t^2, \\ M_{22}(t) &= \frac{4}{5} t, \\ M_{33}(t) &= \frac{16}{5} t. \end{aligned} \tag{20}$$

Using Eqs. (20), the determinants of $M(t)$ and $B(t_1,t_2)$ can now be evaluated leading to

$$\text{Det}\, M(t) = \frac{64}{375} \gamma^2 t^5,$$

$$\text{Det}\, B(t_1,t_2) = \frac{8\gamma^2 (t_2+t_1)^5}{125} \left[4\left\{ \frac{1}{3} - \frac{2t_2^2 t_1}{(t_2+t_1)^3} \right\} - \left\{ 1 - \frac{2t_2^2}{(t_2+t_1)^2} \right\}^2 \right]. \tag{21}$$

The autocorrelation function follows:

$$a(t_1,t_2) = \frac{1}{2} \frac{W^{5/4}(t_1,t_2)}{\left[1 - \frac{3}{4} W^2(t_1,t_2) \right]^{1/2}}, = \frac{1}{2} \frac{\text{sech}^{5/2}(T/2)}{\left[1 - \frac{3}{4} \text{sech}^4(T/2) \right]^{1/2}}, \tag{22}$$

where $W(t_1,t_2)=4t_1t_2/(t_2+t_1)^2$, $T=T_1-T_2$ and the final form follows after introducing the new time variable $T_i = \ln t_i$. After this change of the time variable, the autocorrelation function depends only on the time difference T_1-T_2. Since the process $X(T)$ is also Gaussian, the process $X(T)$ is a Gaussian stationary process. This will be the case whenever the autocorrelation function of X depends on t_1 and t_2 only through the ratio t_1/t_2, i.e., when it exhibits a scaling form.

B. Case $d=2$

For $d=2$, we follow the same analysis as for $d=3$ but with the change of variables

$$(k_x, k_y, t) \to (q_x, q_y - \gamma k_x \tau, \tau). \tag{23}$$

The solution of Eq. (7) in the large-t limit is given by the $d=2$ analog of Eq. (10)

$$m(\vec{k},t) = m(k_x, k_y+\gamma k_x t, 0) \exp\left[-\frac{1}{4} \sum_{a,b=1}^{2} k_a M_{ab}(t) k_b \right]. \tag{24}$$

The matrix elements $M_{ab}(t)$ can be evaluated for large t using asymptotic analysis along the lines outlined in Ref. [14], with the result

$$M_{11}(t) = 4\gamma t^2\sqrt{\ln \gamma t} - \frac{3\gamma t^2}{\sqrt{\ln \gamma t}},$$

$$M_{12}(t) = 4t\sqrt{\ln \gamma t} - \frac{2t}{\sqrt{\ln \gamma t}},$$

$$M_{22}(t) = \frac{4}{\gamma}\sqrt{\ln \gamma t}, \qquad (25)$$

where we have retained just the leading subdominant terms, of relative order $1/\ln(\gamma t)$.

The subleading terms in $M_{11}(t)$ and $M_{12}(t)$ are necessary as there are cancellations to leading terms in the determinant of $M(t)$, which is given by Det $M(t)=4t^2$. Using Eq. (24) the following averages can be calculated:

$$\langle [m(\vec{x},t)]^2\rangle^{1/2} = \left[\frac{\Delta}{2\pi}\frac{1}{\sqrt{\mathrm{Det\,M(t)}}}\right]^{1/2} = \frac{1}{2t}\sqrt{\frac{\Delta}{2\pi}},$$

$$\langle m(1)m(2)\rangle = \frac{\Delta}{(2\pi)}\frac{1}{\sqrt{\mathrm{Det}\,B(t_1,t_2)}}, \qquad (26)$$

where the matrix elements B_{ab} are given by the expressions in Eq. (18) but with the corresponding $M_{ab}(t)$ given by their $d=2$ equivalents in Eq. (25). The autocorrelation function $a(t_1,t_2)$ for $d=2$ can now be evaluated using the set of Eqs. (26) to give

$$a(t_1,t_2) = \left[\frac{4t_1t_2}{t_1^2\left(1+\sqrt{\frac{\ln \gamma t_2}{\ln \gamma t_1}}\right)+t_2^2\left(1+\sqrt{\frac{\ln \gamma t_1}{\ln \gamma t_2}}\right)}\right]^{1/2}. \qquad (27)$$

Note that $a(t_1,t_2)$ given by Eq. (27) does not have a scaling form, i.e., it is not simply a function of t_1/t_2, due to the logarithms. However it does have a scaling *regime*. In the limit $t_1\to\infty$, $t_2\to\infty$, with t_1/t_2 fixed but arbitrary, the ratio of logarithms can be replaced by unity and $a(t_1,t_2)$ depends only on t_1/t_2 in this regime. In terms of the new time variable $T=\ln(t_2/t_1)$, one obtains

$$a(t_1,t_2) = \sqrt{\mathrm{sech}(T)}, \qquad (28)$$

where $T=T_1-T_2$, i.e., the process $X(T)$ becomes stationary in the defined scaling limit. We will use Eq. (28) rather than Eq. (27) to extract θ for $d=2$, but one must note the special limit taken to derive Eq. (28) where the persistence probability $P(t_1,t_2)$ is the probability that a point moving with the flow has stayed in the same phase between times t_1 and t_2.

III. INDEPENDENT INTERVAL APPROXIMATION

The above analysis in both $d=3$ and $d=2$ shows that $X(t)$ is stationary in the new time variable T (with the caveat noted above for $d=2$). We note that the expected form for the probability $P_0(t)$ of $X(t)$ having no zeros between t_1 and t_2, namely, $P_0\sim(t_1/t_2)^\theta$ for $t_2\gg t_1$, becomes exponential decay $P_0\sim e^{-\theta(T_2-T_1)}$ in the new time variable. This reduces the problem of calculating the persistence exponent to the calculation of the decay rates [17].

The order parameter field in the OJK theory is given by $\phi=\mathrm{sgn}(X)$. The autocorrelation function

$$A(T) = \langle\phi(0)\phi(T)\rangle = \langle \mathrm{sgn}X(0)\mathrm{sgn}X(T)\rangle, \qquad (29)$$

for the field ϕ at a space point moving with the flow, is given by

$$A(T) = \frac{2}{\pi}\sin^{-1} a(T), \qquad (30)$$

which follows from the fact that ϕ is a Gaussian field [18]. We will determine the persistence probability $P_0(t)$ from $A(T)$.

We briefly discuss the IIA [1] and use it to obtain approximate values for the exponent θ following the development in Ref. [3]. In the scaling limit, the interfaces occupy a very small volume fraction and as a result $\phi(T)$ takes values ± 1 almost everywhere. The correlator $A(T)$ can be written as

$$A(T) = \sum_{n=0}^{\infty}(-1)^n P_n(T), \qquad (31)$$

where $P_n(T)$ is the probability that the interval T contains n zeros of $\phi(T)$. For $n\geq 1$, $P_n(T)$ is approximated by assuming that the intervals between zeros of X are independent

$$P_n(t) = \langle T\rangle^{-1}\int_0^T dT_1\int_{T_1}^T dT_2\cdots\int_{T_{n-1}}^T dT_n \times Q(T_1)P(T_2-T_1)\cdots P(T_n-T_{n-1})Q(T-T_n), \qquad (32)$$

where $\langle T\rangle$ is the mean interval size, $P(T)$ is the distribution of intervals between successive zeros, and $Q(T)$ is the probability that an interval of size T to the right or left of a zero contains no further zeros, i.e., $P(T)=-Q'(T)$ where the prime indicates a derivative. The IIA has been made in Eq. (32) by writing the joint distribution of zero-crossing intervals as the product of the distribution of single intervals. The Laplace transform of Eq. (32) leads to $\tilde{P}(s)=[2-F(s)]/F(s)$ where

$$F(s) = 1 + \frac{\langle T\rangle}{2}s[1-s\tilde{A}(s)] \qquad (33)$$

and $\tilde{A}(s)$ is the Laplace transform of $A(T)$.

It is straightforward to show that the mean interval size is $\langle T\rangle=-2/A'(0)$. The expectation that $P_0(T)\sim e^{-\theta T}$ for large T implies a simple pole in $\tilde{P}(s)$ at $s=-\theta$. The persistence exponent θ is therefore given by the first zero on the negative axis of the function

$$F(s) = 1 - \frac{s}{A'(0)}\left[1 - \frac{2s}{\pi}\int_0^\infty dT\exp(-sT)\sin^{-1}a\,(T)\right]. \qquad (34)$$

For further analysis it is useful to first extract the asymptotic behavior of the autocorrelation function $a(T)$ of the field $X(T)$. From Eqs. (27) and (22) we find, for $T\to\infty$,

$$a(T) \sim \begin{cases} \exp(-T/2), & d=2, \\ \exp(-5T/4), & d=3. \end{cases} \tag{35}$$

We now turn to the results.

IV. RESULTS

The term $A'(0)$ can easily be evaluated to give $A'(0)=-\sqrt{17/2}/\pi$ in $d=3$ and $-\sqrt{2}/\pi$ in $d=2$. From Eq. (34) $F(0)=1$, and from Eq. (35) $F(s)$ diverges to $-\infty$ for $s\to-5/4$ and $-1/2$ in $d=3$ and 2, respectively. Therefore, the zero of $F(s)$ lies in the interval $(-5/4,0)$ and $(-1/2,0)$ for $d=3$ and 2, respectively. Solving Eq. (34) numerically for this zero, we get the IIA values for the persistence exponent as $\theta=0.5034\ldots$ for $d=3$ and $\theta=0.2406\ldots$ in $d=2$. In the absence of shear the IIA gives [3] $\theta=0.2358\ldots$ in $d=3$ and $0.1862\ldots$ in $d=2$, which agree quite well with simulations [19].

A very interesting feature of the $d=2$ result for θ is that it is exactly twice the value of the exponent obtained within the same approximation (i.e., using OJK theory and the IIA) for the *unsheared* problem in *one* space dimension: $\theta_{\rm sh}^{d=2}=2\theta_{\rm unsh}^{d=1}$. That this must be so is easily seen directly from the form (28) for $a(t_1,t_2)$ for the sheared problem in $d=2$. The equivalent result for the unsheared system in general space dimension is $a(t_1,t_2)=\mathrm{sech}^{d/2}(T/2)$ [3]. For $d=1$ this is identical to Eq. (28) apart from an overall factor 2 in the (logarithmic) timescale T. It follows that the relation $\theta_{\rm sh}^{d=2}=2\theta_{\rm unsh}^{d=1}$ does not require the IIA but only that the underlying field m (or, equivalently, X) be Gaussian, i.e., it requires use of the OJK theory but not the IIA. It is interesting to speculate that it might even hold beyond the OJK approximation, in which case one might imagine that there is a very simple explanation for it. As yet, however, we have been unable to find one.

The autocorrelation function $A(t_1,t_2)$ is also interesting. In the limit $t_2\gg t_1$ that defines the autocorrelation exponent λ [10], via $A(t_1,t_2)\sim(t_1/t_2)^{\lambda}$, the quantity $a(t_1,t_2)$ is small and Eq. (30) can be linearised in $a(t_1,t_2)$ to give, from Eq. (35) with $T=\ln(t_2/t_1)$

$$A(t_1,t_2) \sim \begin{cases} (t_1/t_2)^{5/4}, & d=3, \\ (t_1/t_2)^{1/2}, & d=2. \end{cases} \tag{36}$$

These results give $\lambda=5/4$ for the sheared system in $d=3$, compared to $\lambda=3/4$ in the unsheared system [10], whereas for $d=2$ the autocorrelation exponent takes the same value, $\lambda=1/2$, in both cases. We should repeat the caveat that, for $d=2$, the simple power-law form (36) requires the limit $t_1\to\infty$, $t_2\to\infty$ with t_2/t_1 fixed but large. If $t_2\to\infty$ for fixed t_1, Eq. (27) gives $a(t_1,t_2)\sim(t_1/t_2)^{1/2}[\ln(\gamma t_2)/\ln(\gamma t_1)]^{1/4}$, which does not have a simple scaling form (it is not simply a function of t_1/t_2).

V. CONCLUSION

We have studied the effect of shear flow on the persistence exponent θ, for a system with nonconserved scalar order parameter, using an approximate analytical approach based on the OJK theory and exploiting the "independent interval approximation." The persistence is defined in a frame locally moving with the flow.

The exponent θ is nontrivial and is increased by the presence of shear. This implies that the shear accelerates the change of sign of the fluctuating field. In dimension $d=2$ we find the intriguing result that θ has a value equal to twice that of the unsheared system in $d=1$, within the 0JK theory. The autocorrelation exponent λ increases in the presence of shear for $d=3$ but is unchanged by the shear in $d=2$.

For nonconserved dynamics in the absence of shear, experiments on liquid crystals have been performed to measure both θ [6] and λ [20]. There is also a recent experiment on the measurement of a two-time correlation function in order-disorder phase transition in Cu_3Au [21]. Liquid crystal experiments are a possible candidate for testing our predictions in a model with nonconserved order parameter, and numerical simulations may also prove useful. On the analytical front, the method of the correlator expansion [4] might be used to obtain a more accurate result for θ in $d=3$ than can be obtained using the IIA.

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